Province of the EASTERN CAPE EDUCATION

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Provinsie van die Oos Kaap: Department van Onderwys
Porafensie Ya Kapa Botjahabela: Lefapha la Thuto

NATIONAL SENIOR CERTIFICATE

BUFFALO CITY METRO DISTRICT

GRADE 12

MATHEMATICS P2

PRE-TRIAL EXAMINATION

MARKS : 150
Time : 3 Hours

This question paper consists of 13 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

4. Answers only will NOT necessarily be awarded full marks.

5. Unless stated otherwise, you may use an approved scientific calculator (non-programmable and non-graphical).

6. If necessary, round off answers to TWO decimal places unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. Number the answers correctly according to the numbering system used in this question paper.

9. An information sheet with formulae is included at the end of the question paper.

10. Write neatly and legibly.

QUESTION 1

To celebrate Pi Day at school, learners participate in a competition to write down the value of Pi (π), up to the most correct decimal places. Eleven learners make it to the final round of the competition, where their number of correct decimal places is counted.

The judges stop counting after the first mistake. The results of the eleven learners are shown in the table below.

63	79	50	74	75	66	150	86	72	74	60

1.1 Calculate the:

1.1.1 Mean of the data (2)

1.1.2 The standard deviation for the given data (1)

1.1.3 Number of results that lie outside ONE standard deviation of the mean (3)

1.2 Identify the outlier in the given results. (1)

1.3 The result with the number of the most correct decimal places is increased by k %, while the result with the number of the lowest correct decimal places is decreased by t %. The other nine results remain unchanged.

Only one of the options below correctly reflects the new range of the data in terms of k and t. Choose the answer and write only the letter next to the question number in the ANSWER BOOK

A. $100 + k - t$

B. $150k - 50t$

C. $150k + 50t$

D. $100 + \frac{3}{2}k + \frac{1}{2}t$ (2)

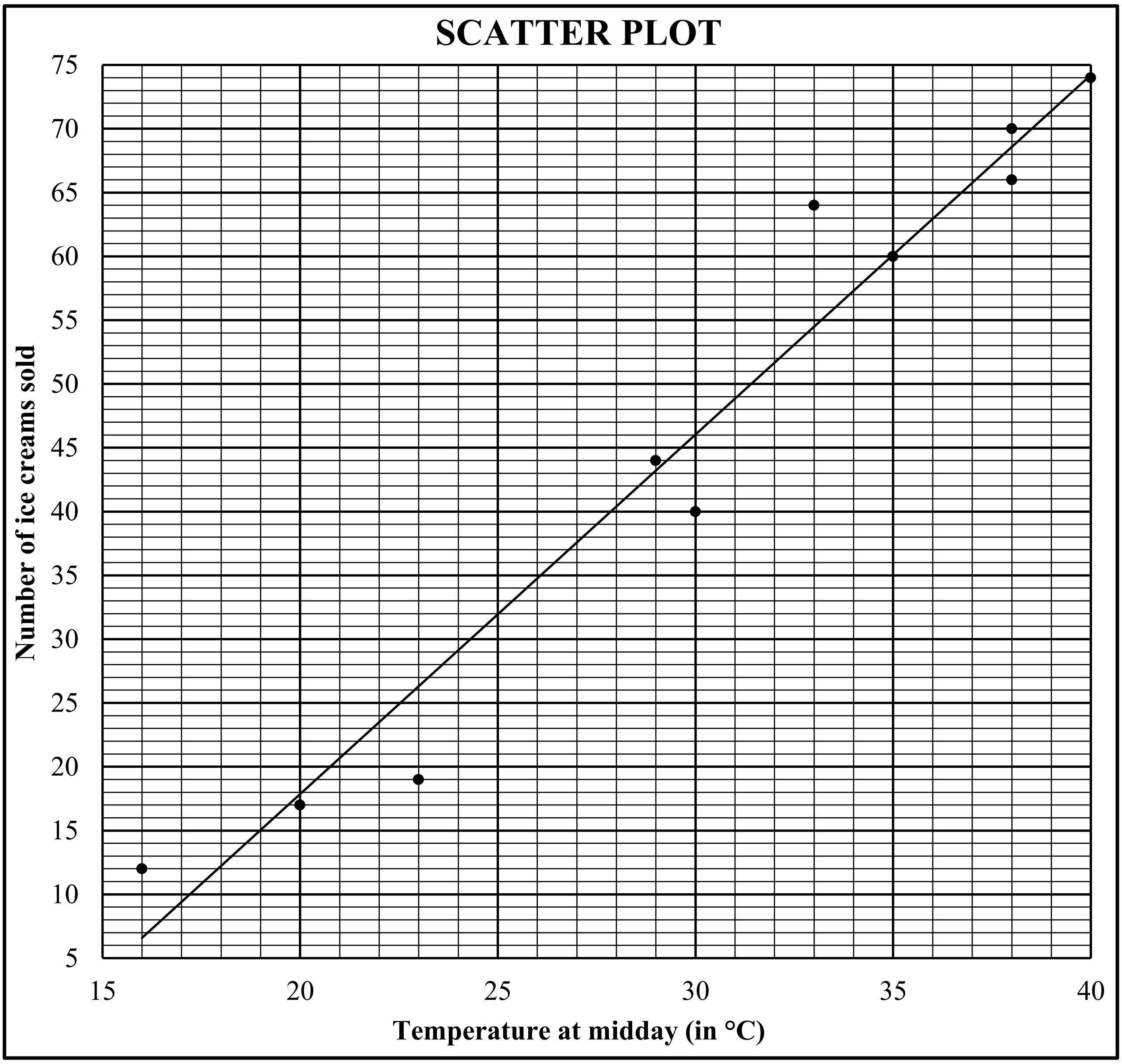
1.4 It was established that a judge made a mistake with one of the six lowest results. The result was corrected and changed to double its original value. How will this change impact the median of the data? Motivate your answer. (2)

[11]

QUESTION 2

On the first Saturday of a month, for a period of ten months, information was recorded about the temperature at midday (in °C); and the number of ice creams sold at an ice cream stand at a particular beach. The data is shown in the table below and represented on the scatter plot. This data's least squares regression line is drawn on the scatter plot.

Temperature at midday (in °C)	16	20	23	29	33	38	40	38	35	30
Number of ice creams sold	12	17	19	44	64	70	74	66	60	40



2.1 Refer to the scatter plot. Would you say that the relationship between the temperature at midday and the number of ice creams sold is weak or strong? Motivate your answer. (2)

2.2 Determine the equation of the least squares regression line. (3)

2.3 Predict the number of ice creams that will be sold on a Saturday if the temperature is 26 °C at midday. (2)

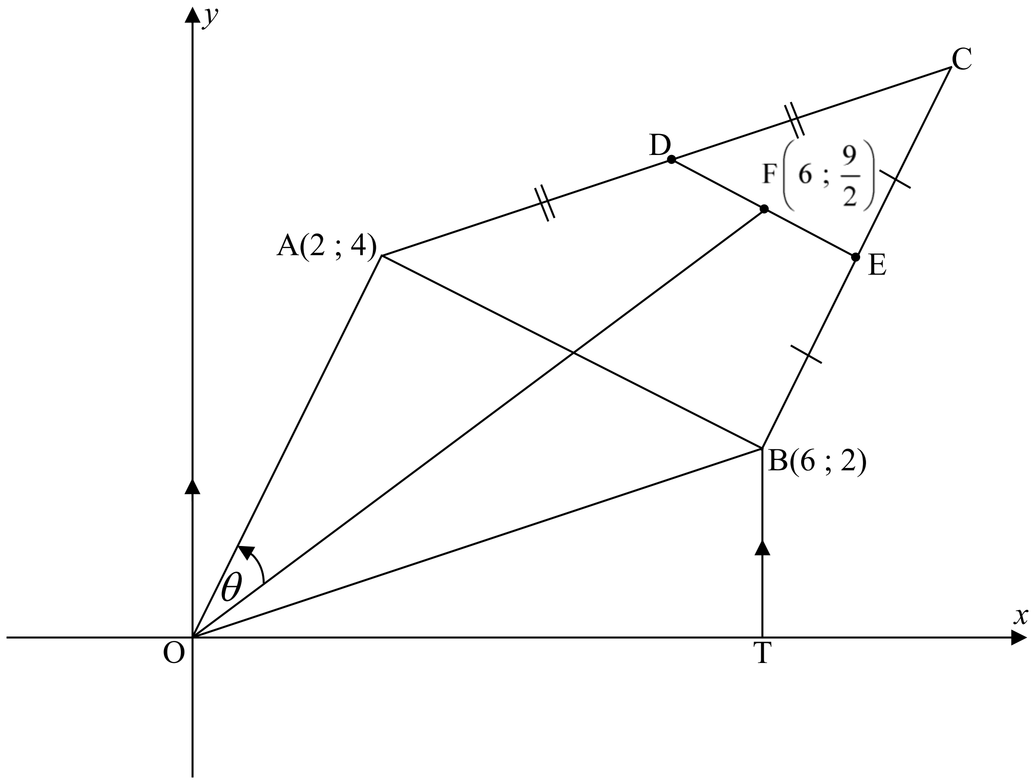
2.4 On another first Saturday of the month, the temperature at midday was 24 °C and 40 ice creams were sold. If this data is added to the data set, how will the prediction of the number of ice creams sold within the given domain be affected? Motivate your answer. (2)

[9]

QUESTION 3

In the diagram below, A(2 ; 4), O, B(6 ; 2) and C are the vertices of a quadrilateral. D and E are the midpoints of AC and BC, respectively. $F\left(6\ ;\ \frac{9}{2}\right)$ is a point on DE.

From B the straight line drawn parallel to the *y*-axis cuts the *x*-axis in T. $A\hat{O}F = \theta$



3.1 Calculate:

3.1.1 The length of AB. Leave your answer in surd form (2)

3.1.2 The gradient of AB (2)

3.2 Prove that $OA \perp AB$ (2)

3.3 Determine the equation of DE. (4)

3.4 Determine the coordinates of C such that AOBC, in this order, is a parallelogram. (3)

3.5 Calculate the:

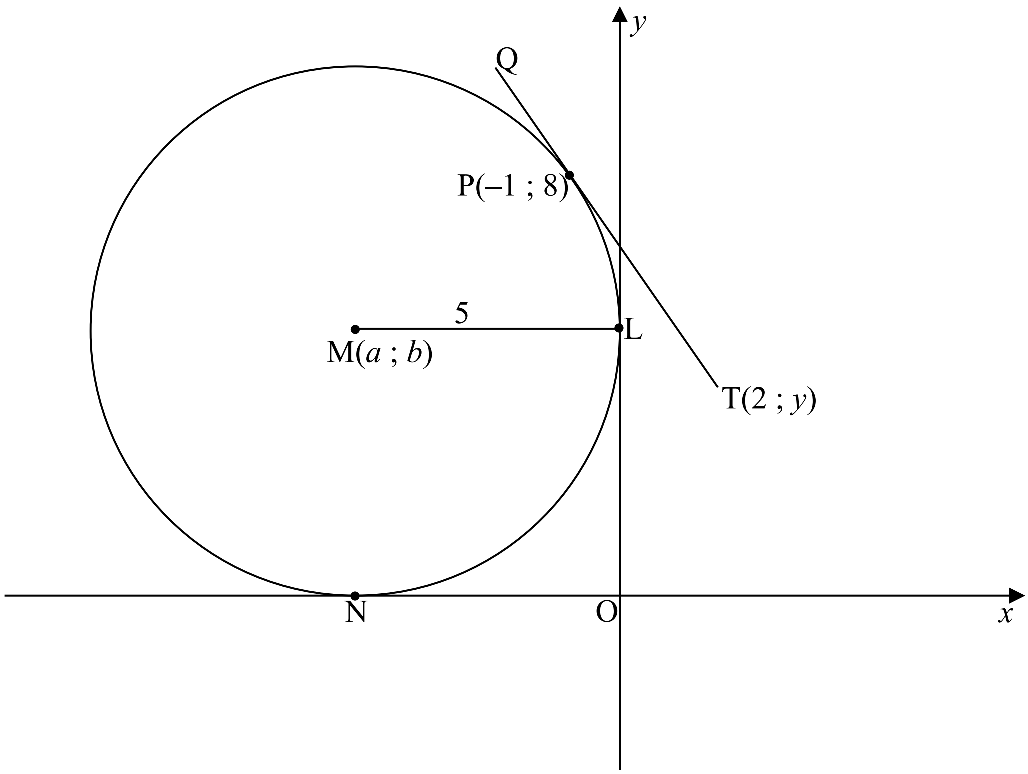
3.5.1 Size of θ (5)

3.5.2 Area of Δ ABT, if A and T are joined to form Δ ABT (4)

[22]

QUESTION 4

In the diagram, a circle centred at M(a ; b) with a radius of 5 units touches the x-axis and the y-axis at points N and L, respectively. QPT is a tangent to this circle at P(–1 ; 8). The coordinates of T are (2 ; y)



4.1 Give a reason why $ML \perp y-\text{axis}$. (1)

4.2 Determine the:

4.2.1 Coordinates of M (2)

4.2.2 Equation of the circle having centre M (2)

4.2.3 Equation of the tangent QPT in the form $y = mx + c$ (5)

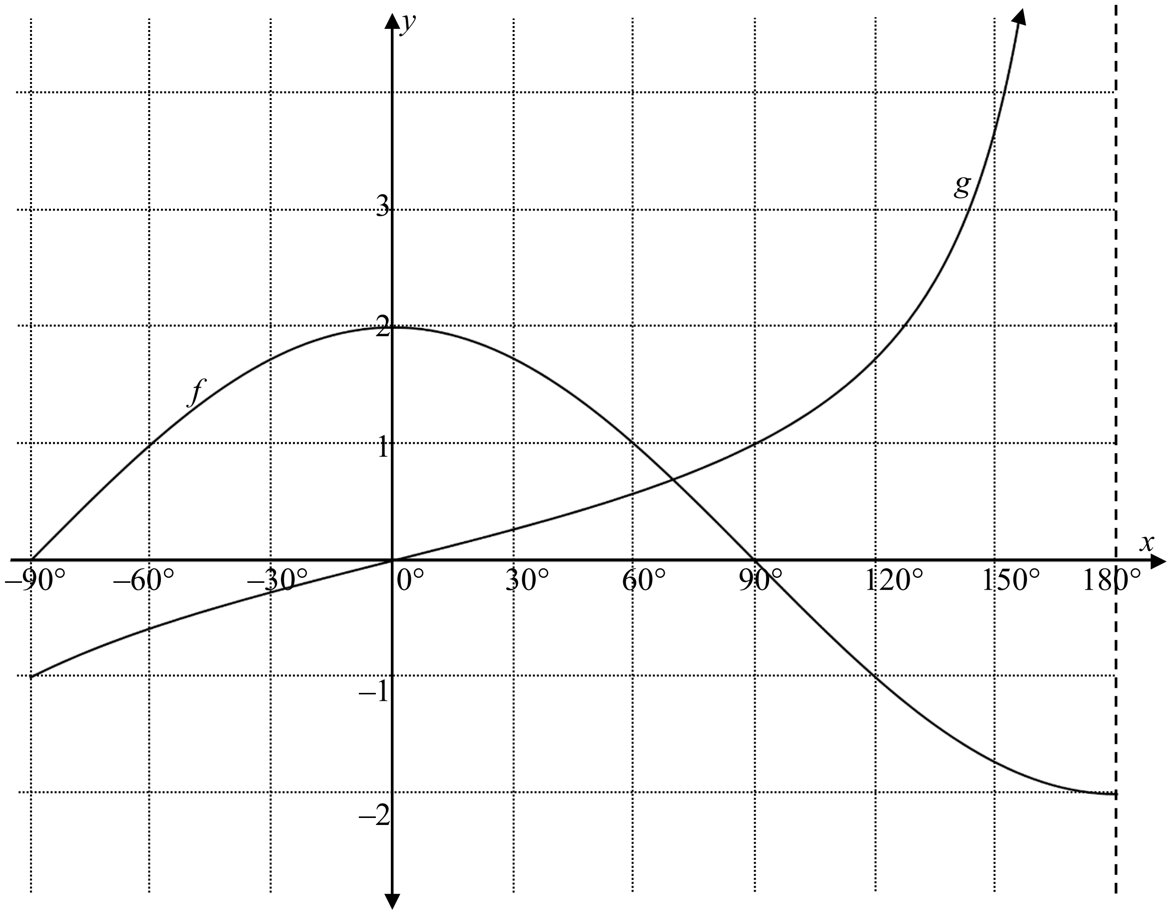
4.3 Another circle, having point T as the centre, touches the circle, having M as the centre externally. Determine the equation of the circle centred at T in the form $(x-h)^2 + (y-k)^2 = r^2$. (6)

4.4 The circle with centre M is translated across the Cartesian plane in such a way that both horizontal and vertical axes remain tangents to the circle simultaneously. Write down all the possible coordinates of the centres of the newly translated circles, given that $\sqrt{xy}$ must be real for ALL values of x and y. (4)

[20]

QUESTION 5

In the diagram below, the graphs of $f(x) = 2\cos x$ and $g(x) = \tan bx$ are drawn for the interval $x \in [-90° ; 180°]$.



Use the graphs to answer the following questions.

5.1 Write down the value of b. (1)

5.2 Write down the range of g for the interval $x \in [-90° ; 180°]$. (2)

5.3 Write down the period of g. (1)

5.4 Write down a value of x, in the given interval, where $g(x + 5°) - f(x + 5°) = 1$. (1)

5.5 Write down TWO values of x in the given interval, where $\dfrac{g(x)}{f'(x)}$ is undefined. (2)

5.6 Write down the value of p if $\sum_{x=0°}^{p} 2\cos x = 0$ (2)

[9]

QUESTION 6

6.1 **WITHOUT using a calculator**, determine the following in terms of sin 25°:

6.1.1 $\sin 335°$ (1)

6.1.2 $\cos 50°$ (2)

6.2 Simplify the following expression to ONE trigonometric ratio:

$$\frac{\sin(-2x).\left(1-\sin^2 x\right)}{\sin(90° + x).\tan x}$$ (6)

6.3 **WITHOUT using a calculator**, simplify $\left(p\tan 30° + q\sin 60°\right)^2$ to a single fraction in terms of p and q. (3)

6.4 Given: $\cos\left(\text{A} - \text{B}\right) = \cos\text{A}.\cos\text{B} + \sin\text{A}.\sin\text{B}$

6.4.1 Use the formula for $\cos\left(\text{A} - \text{B}\right)$ to derive a formula for $\sin\left(\text{A} - \text{B}\right)$. (4)

6.4.2 Prove that $\sin 9\text{A} + \sin\text{A} = 2\sin 5\text{A}.\cos 4\text{A}$ (3)

6.4.3 Write down the maximum value of $3^{2\sin 5\text{A}\cos 4\text{A}}$ (2)

6.5 Determine the general solution of $\cos 2x - 5\cos x - 2 = 0$ (6)

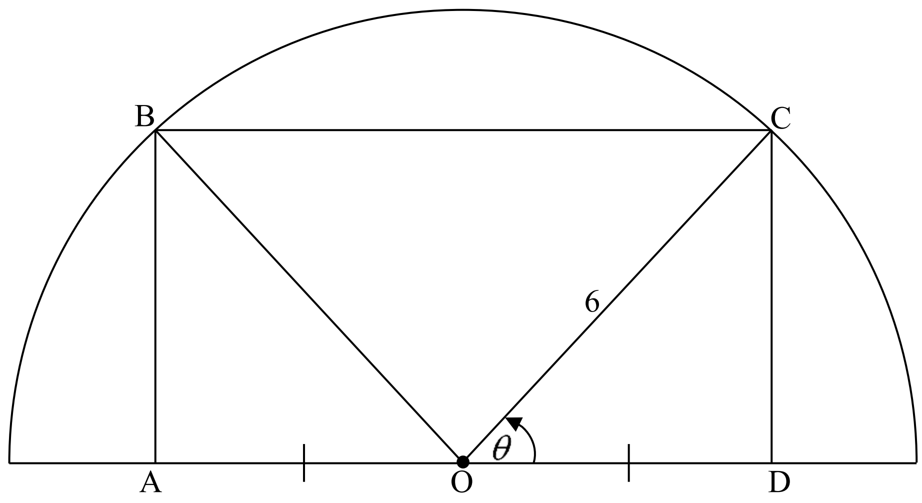
6.6 Given: $\tan x = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots}}}}$, with $x \in \left[0° \,;\, 90°\right)$

WITHOUT using a calculator, show that: $2\sin^2 x = \sin 2x\left(\cos x + 1\right)$ (5)

[32]

QUESTION 7

In the diagram below, O is the centre of the circle. A, B, C and D are points on the semi-circle such that ABCD is a rectangle. The radius of the semi-circle is 6 units, $C\hat{O}D = \theta$ and AO = OD.



7.1 Write $B\hat{O}C$ in terms of θ (2)

7.2 If $\theta = 43°$, calculate the length of BC. (3)

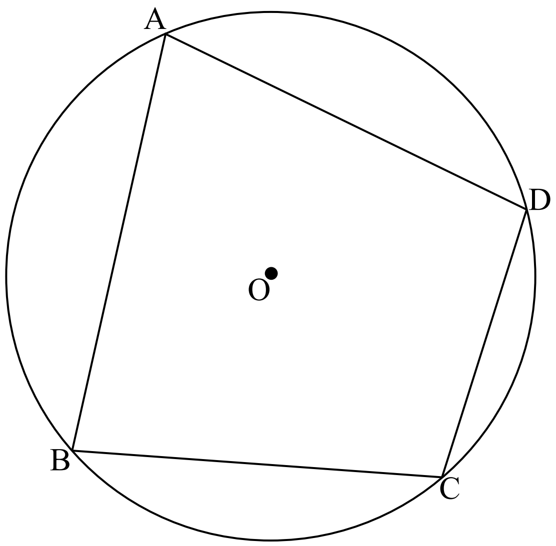
7.3 Points A, B, C and D are shifted along the semi-circle. Calculate the value of θ if ABCD now forms a square. (4)

[9]

Give reasons for your statements in QUESTIONS 8, 9 and 10.

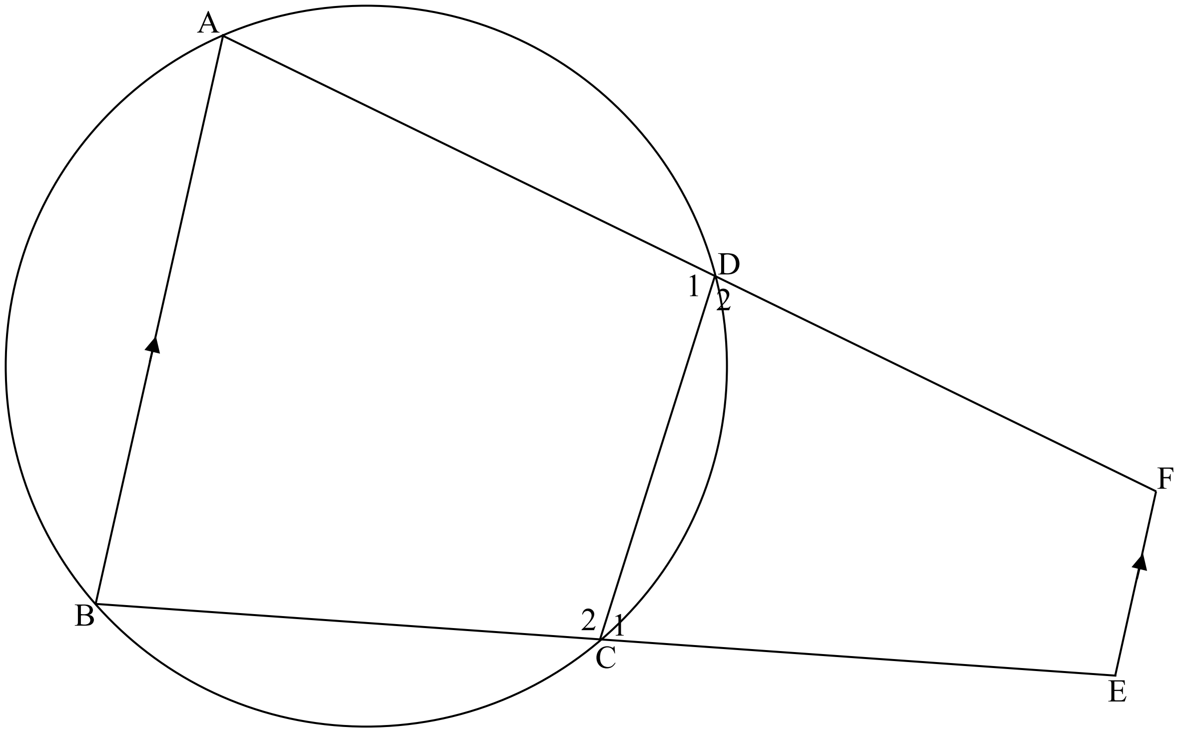
QUESTION 8

8.1 In the diagram, ABCD is a cyclic quadrilateral, and the circle has a centre O.



Prove the theorem which states that $\hat{A} + \hat{C} = 180°$. (5)

8.2 In the diagram below, ABCD is a cyclic quadrilateral. Chords AD and BC are produced to F and E, respectively. F and E are joined such that EF || AB.

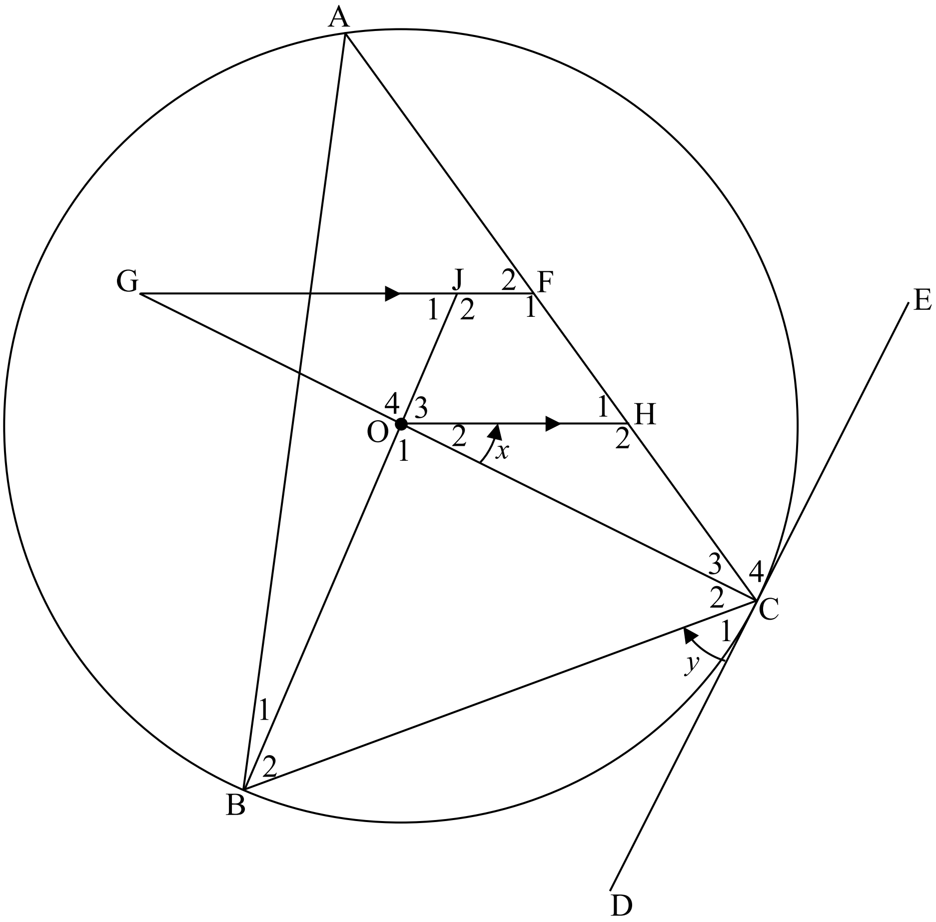


Prove that CEFD is a cyclic quadrilateral. (5)

[10]

QUESTION 9

In the diagram below, O is the centre of the circle, with points A, B and C on the circle. DCE is a tangent to the circle at C. GOC, BOJ, and GJF are straight lines.
F and H are points on AC such that GF || OH.
$\hat{C}_1 = y$, $\hat{O}_2 = x$ and FH : HC = 2 : 3



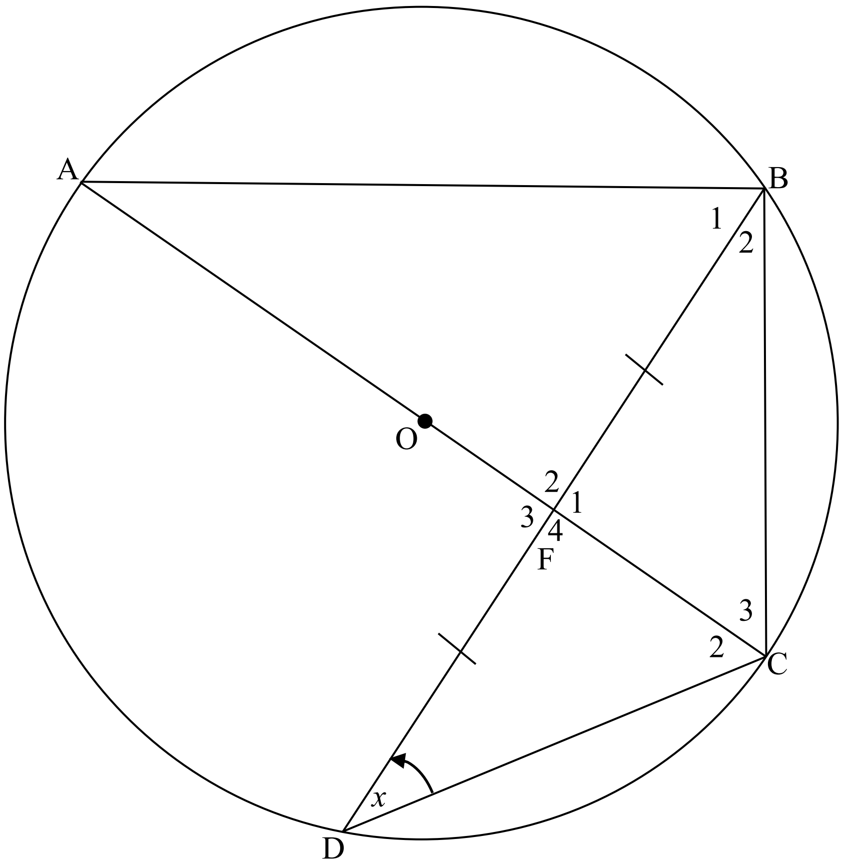
9.1 Calculate, giving reasons, $\hat{J}_1$ in terms of x and y. (6)

9.2 Determine, giving reasons, the value of $\frac{GO}{GC}$. (3)

[9]

QUESTION 10

In the diagram below, A, B, C, and D lie on the circle with centre O.
AOFC and DFB are straight lines, DF = FB and $\hat{D} = x$.



10.1 Determine, with reasons, the size of EACH of the following in terms of x:

10.1.1 $\hat{A}$ (2)

10.1.2 $\hat{C}_3$ (3)

10.2 Prove, giving reasons, that:

10.2.1 $\hat{F}_2 = \hat{F}_3$ (2)

10.2.2 $\Delta CFB \parallel\!| \Delta CBA$ (3)

10.2.3 $DC^2 = FC.AC$ (4)

10.2.4 $\frac{FC}{AC} = \left(1 - \frac{AB}{AO + OC}\right)\left(1 + \frac{AB}{AO + OC}\right)$ (5)

[19]

TOTAL: 150

INFORMATION SHEET:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}\ ;\ r \neq 1$ $\quad S_\infty = \frac{a}{1 - r}\ ;\ -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{n}$ $\quad P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2}\ ;\ \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x - a)^2 + (y - b)^2 = r^2$$

$In\ \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$